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# A SINGULAR VALUE OF $\pi$ .

BY PROF. J. W. NICHOLSON, LOUISIANA STATE UNIV., BATON ROUGE, LA.

ON page 291 of Ray's Calculus may be seen a demonstration of the following well known theorem of Wallis :

$$\frac{\pi}{2} = \frac{2 \cdot 2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8 \cdot 8 \dots}{1 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdot 9 \dots} \quad (1)$$

By the binomial formula

$$(1-1)^n = 1 - n + \frac{n(n-1)}{2} - \frac{n(n-1)(n-2)}{2 \cdot 3} + \frac{n(n-1)(n-2)(n-3)}{2 \cdot 3 \cdot 4} \dots \quad (2)$$

Factoring,

$$(1-1)^n = \frac{(1-n)(2-n)(3-n)(4-n) \dots}{1 \cdot 2 \cdot 3 \cdot 4 \dots} \quad (3)$$

Substituting  $-n$  for  $n$ ,

$$(1-1)^{-n} = \frac{(1+n)(2+n)(3+n)(4+n) \dots}{1 \cdot 2 \cdot 3 \cdot 4 \dots} \quad (4)$$

Multiplying (3) by (4),

$$(1+1)^n(1-1)^{-n} = \frac{(1-n^2)(4-n^2)(9-n^2)(16-n^2) \dots}{1 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 4 \cdot 4 \dots} \quad (5)$$

Substituting  $\frac{1}{2}$  for  $n$ , and reducing,

$$(1-1)^{\frac{1}{2}}(1-1)^{-\frac{1}{2}} = \frac{1 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdot 9 \dots}{2 \cdot 2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8 \cdot 8 \dots} \quad (6)$$

Combining (1) and (6),

$$\pi = \frac{2}{(1-1)^{\frac{1}{2}}(1-1)^{-\frac{1}{2}}}.$$

## ANSWER TO PROF. SCHEFFER'S QUER (P. 31, VOL. VIII.)

BY C. B. SEYMOUR, ATTORNEY AT LAW, LOUISVILLE, KY.

*Query.*—"If of any curve we find the evolute, and of the latter the evolute, and so on ad infin., the ultimate evolute is a cycloid. How is this proved?"

*Answer.*—The proposition stated is not correct.

Let  $s_0$  be the length of the given curve, measured from the origin to any point (the origin being a point on the curve). Let  $\beta_0$  be the inclination of the tangent at that point to the axis of abscissas, and let  $R_0$  be the radius of curvature at that point. Let  $s_n, \beta_n, R_n$  be like quantities for the corres-

ponding points on the  $n$ th evolute of the curve, and  $s_{-n}$ ,  $\beta_{-n}$ ,  $R_{-n}$  like quantities for the corresponding point on its  $n$ th involute. In this notation I regard the centre of curvature at any point as corresponding to that point.

Then

$$R_0 = \frac{ds_0}{d\beta_0}; s_1 = C_0 + R_0 = C_0 + \frac{ds_0}{d\beta_0}.$$

But since the directions of a curve and its evolute at the corresponding points are perpendicular, we have

$$\beta_n = \beta_{n-1} - \frac{1}{2}\pi,$$

and by differentiating,

$$d\beta_n = d\beta_{n-1} = d\beta_0.$$

By the principles of the foregoing argument

$$s_2 = C_1 + \frac{ds_1}{d\beta_0} = C_1 + \frac{d^2s_0}{d\beta_0^2},$$

and generally

$$s_n = C^{n-1} + \frac{d^n s_0}{d\beta_0^n},$$

$C$  with its various subscripts signifying constants.

It is then plain that if  $s_0$  be given as a function of  $\beta_0$ ,  $s_n$  can be at once deduced as a function of  $\beta_0$ , and of course as a function of  $\beta_n$ ; and this function depends on the form of the given function, as the arbitrary constant introduced does not affect the form of the evolute. Thus if  $s_0 = \beta_0^i$  ( $i$  signifying an integer), successive differentiations will at last bring the eq.  $s_n = C_{n-1} \beta_n$ , which is the equation of a circle.

A cycloid does not in general result from taking successive evolutes. If however successive *involute*s be taken, the arbitrary constant introduced by one integration becomes a coefficient of  $\beta_0$  in the next integration; so that the form of the ultimate involute depends on the arbitrary constants.

If in determining the arbitrary constants we make  $\beta_0$  successively zero and  $\frac{1}{2}\pi$  the successive integrations will bring at last an equation indefinitely approximating

$$C + s_{-4n} = C_{-4n} \cos \beta_0 = -C_{-4n} \cos \beta_{-4n}.$$

This is the equation of a cycloid, and no doubt the proposition intended was this:—

If of any curve we find the *involute*, taking the extreme radii of curvature perpendicular to each other, and of the latter the involute in like manner, and so on ad infinitum, the ultimate involute is a cycloid.